JOURNAL OF AIRCRAFT
Vol. 48, No. 2, March–April 2011

Optimal Scheduling and Speed Adjustment in En Route Sector for Arriving Airplanes

Ta-Chung Wang* and Yen-Ju Li†
National Cheng Kung University, Tainan 701, Taiwan, Republic of China

DOI: 10.2514/1.C031222

Flow control is one of the major problems for busy airports during rush hours. Air traffic controllers must quickly arrange the approaching sequence of the aircraft and make necessary separation arrangements. This paper presents a scheduling algorithm to generate the optimal speed-change advisories that minimize both the time to land a group of aircraft and the changes of scheduled time of arrival from estimated time of arrival of each aircraft in this group. A sequence matrix and a separation-time matrix are used to construct the constraints. Comparing with traditional formulations in which the number of constraints grows with the square of the number of aircraft, the number of constraints in the proposed formulation grows linearly with the number of aircraft. The speed-change capabilities for different aircraft are also considered in the constraints. Using the proposed optimization process, the final results will meet both optimal sequence and minimal changes of scheduled time of arrival from estimated time of arrival requirements. Finally, historical data for Taiwan Taoyuan International Airport are used to demonstrate the effectiveness of our approach.

Nomenclature

$\tilde{e}_j$ = estimated time of arrival of the jth aircraft, s
l_j = separation time between the jth and the $(j+1)$th aircraft, s
N = number of aircraft to be scheduled
p_j = latest allowable landing time of the jth aircraft, s
r_j = earliest allowable landing time of the jth aircraft, s
S = sequence matrix
T_{sep} = separation-time matrix of the relationship between all aircraft, s
$v_{l(j)}$ = maximum possible slow down velocity of the jth aircraft, kt
$v_{u(j)}$ = maximum possible speed up velocity of the jth aircraft, kt
Δt = controlling time-window length, s

I. Introduction

WHEN an aircraft arrives at an airport, air traffic controllers (ATCs) ideally guide the aircraft to the correct runway and create enough separation between aircraft. In the case of multiple aircraft, ATCs need to schedule the landing priority. As air traffic increases, work load and pressure on ATCs increase, which may increase the probability of human mistakes. According to the prediction of International Air Transport Association [1], worldwide civil aviation traffic is expected to grow annually at rates between 1 and 6% from 2010 to 2013. The sequencing strategy used by almost all major airports of the world is the first-come, first-served (FCFS) method [2]. Using the FCFS approach, the sooner the aircraft arrives at an airport, the higher landing priority it gets. Although it is easy to implement, the FCFS method has been shown to be inefficient with regard to the use of airport capacities [3]. Therefore, civil aviation authorities all over the world are seeking an automatic decision process for aircraft scheduling, to alleviate the possible dangers inherent in air traffic control. In 1989, the International Civil Aviation Organization (ICAO) suggested developing a new communication, navigation, surveillance, and air traffic management system to promote air traffic safety, decrease delays, and increase the air capacity in anticipation of future growth of air traffic.

In the United States, a system has been under development by the NASA Ames Research Center. This system, referred to as the Center/TRACON (Terminal Radar Approach Control) Automation System (CTAS), is composed of the Final Approach Spacing Tool (FAST), the Traffic Management Advisor (TMA), and the Descent Advisor (DA). FAST provides terminal area controllers with heading and speed advisories to help in generating an accurately spaced flow of aircraft onto the final approach course [4]. TMA generates runway assignments, landing sequences, and landing times for all arriving aircraft. DA generates clearances for en route controllers to handle arrival flows to metering gates. The advisories generated by these tools assist controllers in handling arriving aircraft starting at about 200 n mile from the airport and continuing to the final approach fix [5].

In Europe, a similar system is being implemented by the DLR, German Aerospace Center [6]. This system, referred to as 4D Cooperative Arrival Manager (4D-CARMA), will provide advisories to air traffic controllers. The advisories, which contain the information for landing runways, flight speed, flight altitudes, and flight trajectories, will assist controllers in handling arrival traffic. Controllers can also input their desired sequence to make the system provide the necessary advisories. This system has been simulated in the Frankfurt Airport.

The two systems referenced above control the aircraft in the terminal area. If the airport capacity is greater than the arrival flow, then ATCs can deal with the traffic efficiently. However, if the number of arriving aircraft is greater than the airport capacity, some of these aircraft may need to wait in the air waiting for their respective landing time-slots scheduled by ATCs, and the longer period in the air may pose possible dangers. To solve this problem, the proposed method adjusts the speeds of aircraft in the en route sector, which reduces the possibility of congestion in the terminal area.

II. Preliminaries

A. Previous Research

In this section, we list several previous research in the literature on the two major components of an air traffic management system. The first component is the conflict resolution system and the second component is the aircraft landing scheduling system.

1. *Conflict Resolution*

In air traffic control, a conflict is declared when two aircraft fail to maintain a appropriate separation time or distance between

Received 31 August 2010; revision received 27 November 2010; accepted for publication 27 November 2010.

*Assistant Professor, Institute of Civil Aviation, No. 1 University Road.
†Graduate Master Student, Institute of Civil Aviation.

themselves. The purpose of a conflict resolution system is to generate suitable actions for an aircraft to avoid possible danger. Dugail et al. [7] considered the idea that two intersecting flows of aircraft must avoid each other. Considering avoidance maneuvers modeled by instantaneous heading changes, it has been demonstrated that the required angle of heading changes remain bounded under decentralized, sequential conflict avoidance rules. Mao et al. [8] generated the protected zone by using the relative position and velocity of aircraft. If an aircraft enters the protected zone of another aircraft, the lower priority aircraft should offset its heading vector in order to avoid conflicts. Krozel and Peters [9] used conflict probability maps to detect conflicts. This conflict probability map is generated using either Monte Carlo simulations or analytical approaches. The space around a central aircraft is partitioned into grids. Normal distributions are used to model position, velocity, and heading variables of the surrounding aircraft in each grid. Then the probability of conflict between the central aircraft and the surrounding aircraft can be computed by calculating the probability of the closest distance between the central aircraft and the surrounding aircraft being less than the required separation distance. The positioning errors of the Global Positioning System are also used to calculate the probability of conflict for this aircraft. When a conflict is detected, a conflict resolution scheme analyzes the geometry of all possible maneuvers and the direct operation costs for these maneuvers in order to find the best resolution approach. In this paper, we assume the aircraft on each approaching airway being properly separated. However, conflicts may occur when aircraft coming from different airways merge at the landing airport. The estimated time of arrival (ETA) of each aircraft will be used to check whether conflicts will occur. If conflicts are detected, the proposed approach generates the speed advisories to the aircraft to resolve these conflicts.

2. *Aircraft Landing Scheduling*

When aircraft travel in the air, trailing vortices are generated behind each aircraft. These may pose danger to the trailing aircraft during the landing procedure. Therefore, a proper separation is required between successive landing aircraft. Different aircraft types generate different amounts of trailing vortices and have different capabilities to maintain a stable flight under different amounts of unstable airflow. Hence, the separation between different types of aircraft are different. Moreover, a change in the landing sequence affects the throughput of an airport and the inherent delays of improper landing sequence may generate excessive amount of costs to airlines. Therefore, the aviation industry looks to optimal landing sequence generation as a potential source of considerable savings.

Abela et al. [10] considered a static case of the landing problem. In the static case, all aircraft present at the same time instant waiting for landing are considered. Considering the fact that each aircraft need to be specified exactly one landing time to land and different combinations of landing sequences will have different separation times accumulated, the aircraft landing sequencing problem is shown to have a structure similar to the traveling-salesman problem [2]. Using this kind of formulation, the number of constraints grows with the square of the number of aircraft, and the time used for finding the solution grows rapidly accordingly. Hence, heuristic methods are used to get suboptimal solutions. Beasley et al. [3,11] and Bianco et al. [2] later expanded Abela et al.'s [10] formulation to the dynamic case with the additional consideration of landing time window. In the dynamic case, some new aircraft may enter the system and some existing aircraft may leave the system. The mixture of different aircraft types hence changes over time. To avoid repeatedly solving the whole scheduling problem, the technique of using the maximum position-shifting constraint is used [2]. Helmke et al. [6] dealt with the successive application of time-based arrival management and automatic arrival–departure coordination. Helmke et al.'s approach sets aircraft positions, the calculated arrival intervals and the controller inputs as the constraints. Then a heuristic tree search algorithm is used to find the optimal sequence. Helmke et al. [6] and Beasley et al. [3,11] both used heuristic method to find a suboptimal result, but Helmke et al.'s [6] approach also provides the trajectory advisories for all the arriving aircraft. Following the trajectory advisories, these aircraft can land with reduced landing time delays while satisfying separation regulations. Zhang et al. [12] found an optimal sequence from multiple approach routes. The purpose of this sequence was to find the minimum time to land all of the aircraft on all of the approach routes. A genetic algorithm was used to search for the optimal sequence after considering the separation between aircraft. Zhang et al.'s formulation is similar to Beasley et al.'s [3,11] approach. The definition for separation constraints, time-window constraints, and position constraints are similar, but they use different heuristic methods to find the solution.

A different approach was proposed by Resmerita and Heymann [13]. They presented a framework for controlling the spacing between approaching aircraft using an airspace-partitioning technique. The airspace is divided into several parts, and no part can be occupied by more than one aircraft at a time. Each aircraft has its path to the destination airport. Conflicts will occur when two aircraft enter the same airspace. In this case, the aircraft that has a lower priority will be directed to wait until the other aircraft has left the airspace. Resmerita and Heymann's approach uses airspace separation, but the separation distance may not be properly adjusted. Therefore, the limited airspace may not be properly used.

Lin and Jian [14] used the separation constraints between aircraft along with the runway constraints to calculate the separation time. After this calculation is completed, each aircraft receives an updated value of its landing time. Using this reassigned time, a flight speed that does not cause conflict with other aircraft can then be calculated. This research is different from other research. It develops its own regulation to find the optimal sequence and to control an aircraft's velocity on the course to achieve the optimal arrival time.

For parallel approaches, Kupfer [15] used an optimization model for a scheduling problem for closely spaced parallel approaches. It takes temporal, pairing, sequencing, separation routes and grouping constraints into account. Kupfer evaluated the performance differences between the computation of solutions using mixed-integer linear programming and genetic algorithms. However, the formulation still suffers from the problem of square growth in the number of constraints, and hence a heuristic approach is inevitable.

Comparing with existing methods, the proposed formulation used a sequence matrix as the decision variable that reduces the number of constraints required as compared with traditional formulations. Moreover, the proposed method controls the aircraft in the en route sector, which can alleviate the aircraft's density around the airport. Other than the capability to provide a optimal sequence of the arriving aircraft, the proposed approach provides simple speed advisories to the arriving aircraft that minimizes both the time to land all the aircraft and the changes of scheduled time of arrival (STA) from ETA.

B. Proposed Approach

This research provides an automatic landing sequence generator for arriving aircraft. The ETA for all aircraft are used to check if the separation time is adequate between these aircraft. If conflicts are detected, an optimal scheduling algorithm is used to resolve these conflicts. This optimization process has two objectives. The first objective is to minimize the *operation time*, which is the time interval required between the STA of the first and the last aircraft. The second objective is to minimize the changes of STA from ETA of each aircraft. Following the velocity advisories generated by the proposed algorithm, we could achieve both the minimum changes of STA from ETA and minimum operation time for landing all the aircraft.

A key contribution of this research is in the formulation of the separation-time constraints. A separation-time matrix and a sequence matrix are used to express the necessary separation constraints. Using the proposed formulation, N aircraft will have $N-1$ constraints, as opposed to traditional formulations that need $N(N-1)$ separation constraints in their formulations. The N-square growth in constraints greatly increases the time required to find an optimal solution. Therefore, in many present studies, heuristic algorithms such as genetic algorithms [10,12] and tree search algorithms [6] are used in order to generate the solution within an acceptable amount of computation time. Since heuristic solutions are not guaranteed to be optimal, the answer may not give ATCs the best advisory. In our formulation, the number of constraints grows linearly with the number of aircraft. Therefore, heuristics algorithms are no longer required to find the solution within an acceptable amount of time. Moreover, our formulation also considers the aircraft landing before the aircraft being scheduled. With the additional consideration of maximum position-shifting number, the proposed formulation also works in a dynamic case.

III. Background Knowledge

A. Separation Adjustment Methods

Aircraft may approach an airport along several different airways, and these airways will merge at some points before reaching the airport. Consider the airways around Taiwan Taoyuan International Airport as an example. Figure 1 shows the approach airways for this airport. Aircraft approaching the airport from the north via airways A1, B576, or R595 will merge at waypoint SEPIA then move onto waypoint AUGUR. According to [16], it is the ATCs' responsibility to provide enough separation between aircraft using radar vector changes, speed control, or holding in the holding patterns. For this reason, as can be seen in this figure, several holding patterns shown as racetrack patterns are designed at these merging waypoints. These holding patterns provide alternative routes for incoming aircraft in order to delay their time to enter these merging points.

1. Separation Using Holding Patterns

The main purpose of a holding pattern is to allow aircraft to circle around until the situations are such that they can proceed without any danger. In most cases, a holding pattern is compelled by one of the following two situations. One situation is if an airport is closed, and no aircraft can land or take off. The other is a situation in which the number of arriving aircraft may be so huge that other methods cannot control the arriving aircraft efficiently. The vertical separation in a holding pattern is 1000 ft. As shown in Fig. 2, if the first aircraft maintains at 5000 ft in the holding pattern, the second aircraft should maintain at 6000 ft, and so on.

2. Separation Using Radar Vectors

Other than using holding patterns, ATCs can also use route changes to generate proper separations between aircraft. As shown in Fig. 3, if there is not enough separation between aircraft A and aircraft B when these two aircraft arrive at the airport, ATCs can make aircraft A pass though route 1 and aircraft B pass though route 2. Since the approaching route for aircraft B is longer than the approaching route for aircraft A, the separation between aircraft A and aircraft B can be increased accordingly.

3. Separation Using Speed Adjustments

Speed control is a standard separation method currently being used in which ATCs control the speed of arriving aircraft. For example, in Fig. 3, ATCs can make aircraft A speed up or aircraft B slow down, and then the arrival time interval between aircraft A and aircraft B becomes adequate.

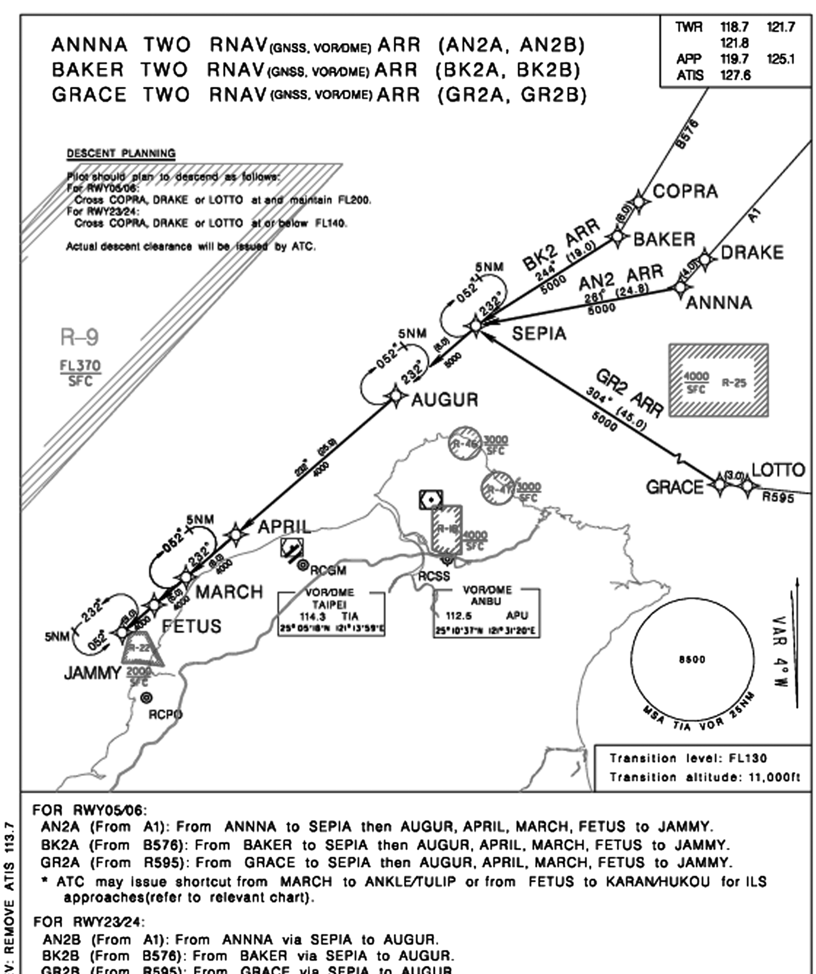

Fig. 1 Approaching airways of the Taiwan Taoyuan airport [20].

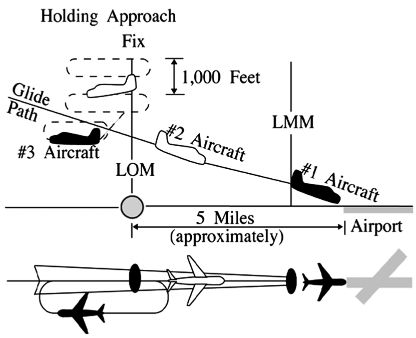

Fig. 2 Separations using holding patterns [21].

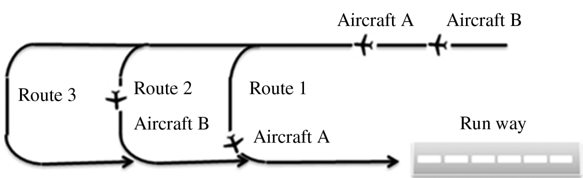

Fig. 3 Separation using radar vectors.

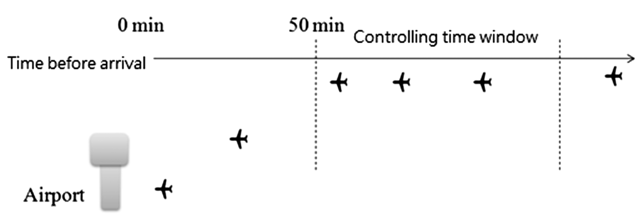

Fig. 4 Controlling time window.

In the proposed algorithm, we adjust the speeds of aircraft in a time window located in the en route sector. This time window is referred to as the *controlling time window* in this paper. All the arriving aircraft entering the controlling time windows will be considered. Figure 4 illustrates the idea of the controlling time window of the proposed algorithm. The aircraft that flies before the fist aircraft in the controlling time window is defined as the boundary aircraft and is labeled as the 0th aircraft. The boundary aircraft will not be rescheduled but will be included in the problem formulation, since it still affects the arrival time of the first rescheduled aircraft. From recorded live traffic data, most of the aircraft start descending at about 50 min before arriving at the airport. Therefore, we set the beginning of the time windows at 50 min before the ETA. The length of the time window may be varied, and different lengths of time window have different effects. As will be shown in this paper, longer time windows provide more flexibilities in rearranging the arriving aircraft. However, making the time window too long may increase the problem size, since more aircraft are being considered. On the other hand, if the time windows are too short, speed control may not be able to provide a feasible solution. In this case, holding patterns or radar vector changes are required in order to maintain proper separations between aircraft. The length of the time window can also be made different for each aircraft to increase the flexibility of the proposed algorithm. For simplicity, the same controlling time-window length for all aircraft will be considered in this paper.

B. Separation Regulations and Conflict Detection

1. Time-Based Separation

The separation requirements for landing aircraft can be time-based or distance-based. These separation requirements are the combination of the effect of trailing vortices and surveillance errors. Distance-based separation standards are currently used, since the separation conditions can be easily identified and checked by ATCs on their radar displays. However, comparing with time-based separation, distance-based separation is known to be less efficient when considering different weather conditions [17] and the usage of airport capacities [18]. Hence, time-based separation is commonly proposed in automatic landing sequencing algorithms [2,3,6,10,11,17,18] and will be used in the proposed algorithm.

Before applying the proposed algorithm, ATCs should first know whether there is necessity to reschedule the incoming aircraft or not. The ETA of all aircraft in the controlling time window can be used to check if conflicts will occur in the future. This can be done by considering the spatial separation distance classified by ICAO regulations [19]. This regulation is designed according to the maximum takeoff weight (MTOW) of each aircraft. The definition of each aircraft's type is listed in Table 1.

Because the turbulence generated by the leading aircraft may destabilize the trailing aircraft, a proper separation time between aircraft is very important. For different combinations of the types of aircraft, different separation times are applied. ICAO's separation regulation is originally defined by distance. Using the average landing speed of 136 kt as the reference speed [17], ICAO's regulations can be converted to time-based separation regulations. Table 2 shows the separation times between different types of aircraft according to ICAO's regulations.

2. Conflict Detection

Before applying the optimization process, we need to know how many aircraft require rescheduling. If there is no conflict between the approaching aircraft, no further rearrangements are required. We can check whether conflicts will occur in the future using the information listed in Tables 1 and 2. Let $\bar{l}_j$ be the minimal separation time between the jth aircraft and the $(j+1)$th aircraft. Then we calculate the accumulated separation time between the first aircraft and the Nth aircraft and check if the ETA difference between the first aircraft and the Nth aircraft accommodates these separation times. That is,

$$\tilde{e}_N - \tilde{e}_1 \geq \sum_{j=0}^{N-1} \bar{l}_j \tag{1}$$

where $\tilde{e}_i$ is the ETA of the ith aircraft and $\bar{l}_j$ can be obtained from Table 2. If the separation time is insufficient or the above inequality cannot be satisfied, these aircraft should be rescheduled.

IV. Finding the Optimal Sequence

A. Single-Runway Formulation

1. Objective Function

Our first target is to minimize the operation time. If there are N aircraft to be rescheduled, the cost function is defined as

Table 1 Classification of the aircraft

MTOW	Classification
MTOW > 136 t	Heavy
136 t > MTOW > 6 t	Medium
6 t > MTOW	Small

Table 2 Separation time between aircraft

	Follower			
Leader	Heavy	B757	Medium	Small
Heavy	106 s	132 s	132 s	159 s
B757	106 s	106 s	106 s	132 s
Medium	79 s	79 s	79 s	79 s
Small	79 s	79 s	79 s	79 s

$$F_1 = \sum_{j=0}^{N-1} l_j \quad (2)$$

where F_1 collects the separation time between each pair of consecutive landing aircraft. Hence, F_1 represents the total time required for these aircraft to land.

2. Constraints

The constraints used in formulating the single-runway scheduling problem include performance constraints, separation-time constraints and sequence matrix constraints.

Performance constraints describe that the STA of the jth aircraft should be located within the arrival time windows between r_j and p_j. These performance constraints are

$$e_{j+1}^T Sr \le \tilde{e}_0 + \sum_{i=0}^{j-1} l_i \quad \text{for } j = 1, \ldots, N \quad (3)$$

and

$$\tilde{e}_0 + \sum_{i=0}^{j-1} l_i \le e_{j+1}^T Sp \quad \text{for } j = 1, \ldots, N \quad (4)$$

where r, p and $\tilde{e}$ are the vectors formed by r_j, p_j and $\tilde{e}_j$ respectively and e_{j+1} is the usual basis vector. $S \in \Re^{(N+1)\times(N+1)}$ is the sequence matrix whose elements are defined as follows: If the $(j-1)$th aircraft is scheduled as the $(i-1)$th aircraft to land, then $S_{ij} = 1$; otherwise, $s_{ij} = 0$. The initial index j of each aircraft can be arbitrarily chosen, except that $j = 1$ is always assigned to the boundary aircraft. For example, if the original sequence of landing is ABC and the optimal landing sequence is BCA, S becomes

$$S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad (5)$$

Note that s_{11} will be enforced to be 1, since the boundary aircraft always lands before all the aircraft in the controlling time window. Although the boundary aircraft will not be rescheduled, its position and type still affect the scheduling result.

The separation-time constraints ensure that the separation time between two adjacent aircraft conforms to the rule listed in Table 2. These constraints can be formulated as

$$e_{j+2}^T S T_{\text{sep}} S^T e_{j+1} \le l_j \quad \text{for } j = 0, \ldots, N-1 \quad (6)$$

where $T_{\text{sep}} \in \Re^{(N+1)\times(N+1)}$ is the separation matrix whose elements $T_{\text{sep}(ij)}$ represent the separation time when the $(i-1)$th aircraft is scheduled behind the $(j-1)$th aircraft. Note that using the sequence matrix as the decision variable, in this $N+1$ aircraft case, we are now able to use only N constraints instead of $N(N+1)$ constraints [2,3,6,10–12].

The constraints of the sequence matrix S ensure that there is only one nonzero element in each column and row of S, i.e.,

$$\begin{cases} s_{11} = 1 \\ s_{i1} + s_{i2} + \cdots + s_{i(N+1)} = 1 \\ s_{1i} + s_{2i} + \cdots + s_{(N+1)i} = 1 \\ s_{ij} \in \{0, 1\} \end{cases} \quad \text{for } i = 1, \ldots, N+1 \quad (7)$$

where the first equality constraint ensures that the sequence of the boundary aircraft is unaffected.

B. Parallel-Runway Formulation

Most busy international airports have two or more runways. If an airport has multiple runways, ATCs should make the separations between aircraft and provide suitable runways for them. According to [19], if the centerlines of the runways are spaced by not less than 1525 m, aircraft approaching to each runway can be considered

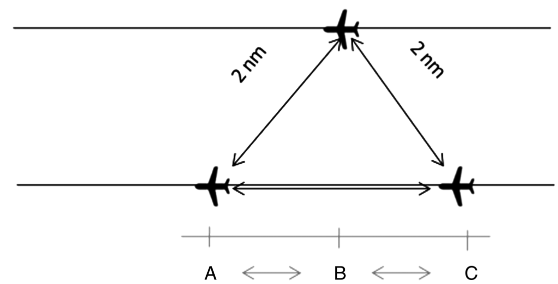

Fig. 5 Separation distance between aircraft.

independently. This centerline distance requirement can be reduced if proper navigation equipments are provided. For runways whose centerline distance does not meet independent operation requirements, dependent parallel approach should be considered. In this case, diagonal separation between aircraft should also be considered. For example, as shown in Fig. 5, if the runway centerlines are spaced by less than 1525 m, the separation distance interval between the aircraft is 2 n mile. That is, a minimum of 2 n mile radar separation diagonally between successive landing aircraft on adjacent courses should be enforced.

In the parallel-approach case, the objective function remains the same. That is,

$$F_1 = \sum_{j=0}^{N-1} l_j$$

For constructing the constraints, the sequence matrix used in single-runway formulation can be augmented to the parallel runway case. Under the performance constraints, Eq. (3) can be rewritten as

$$e_{j+1}^T Yq \le e_1^T Yg + \sum_{i=0}^{j-1} l_i \quad \text{for } j = 1, \ldots, N \quad (8)$$

The definition of Y is as follows:

If the $(k-1)$th aircraft is scheduled as the $(i-1)$th aircraft to land on runway L, then $y_{i(2k-1)} = 1$; otherwise, $y_{i(2k-1)} = 0$.

If the $(k-1)$th aircraft is scheduled as the $(i-1)$th aircraft to land on runway R, then $y_{i(2k)} = 1$; otherwise, $y_{i(2k)} = 0$. For example, if we set aircraft B and aircraft A to land on runway L, and aircraft C lands on runway R, and the landing sequence is BCA, then

$$Y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

In Eq. (8), $g = \text{kron}(\tilde{e}^T, \mathbf{1}_{2\times1})$, which is the Kronecker product of $\tilde{e}^T$ and $\mathbf{1}_{2\times1}$. g describes the ETA of each aircraft landing on different runways. Using g, we assume that an aircraft can reach both runways at the same time. $q = \text{kron}(r^T, 1_{2\times1})$ means the earliest allowable arrival time for arriving aircraft.

Similarly, Eq. (4) can be rewritten as

$$e_1^T Yg + \sum_{i=0}^{j-1} l_i \le e_{j+1}^T Yu \quad \text{for } j = 1, \ldots, N-1 \quad (10)$$

Here, $u = \text{kron}(p^T, 1_{2\times1})$ means the latest arrival time for arriving aircraft. Again, Eqs. (8) and (10) are the constraints that the STA for each aircraft should be located in each aircraft's arrival time window.

The separation-time constraints (6) can be rewritten as

$$e_{j+2}^T Y \tilde{T}_{d\text{sep}} Y^T e_{j+1} \le l_j \quad \text{for } j = 0, \ldots, N \quad (11)$$

where $\tilde{T}_{d\text{sep}}$ means the separation-time relationship between each aircraft landing on different runways. Let $\bar{T}_{d\text{sep}} = \text{kron}(T_{\text{sep}}, 1_{2\times2})$. Then the elements of $\tilde{T}_{d\text{sep}}$ are

$$\begin{cases} \tilde{T}_{d\text{sep}(2i-1,2j)} = \tilde{T}_{d\text{sep}(2i,2j-1)} = 3600 \times \dfrac{\sqrt{2^2 - d^2}}{v_r} \\ \tilde{T}_{d\text{sep}(2i-1,2j)} = \bar{T}_{d\text{sep}(2i-1,2j-1)} \end{cases}$$
$$\text{for } i, j = 0, \ldots, N \tag{12}$$

where d (in nautical miles) is the distance between the centerlines of the parallel runways, and v_r (in knots) is the reference landing speed.

Constraint (11) contains the separation-time requirements with adjacent landing aircraft. We should also consider the separation time between the jth aircraft and the $(j-2)$th aircraft if the $(j-1)$th aircraft lands on a different runway, whereas the jth and the $(j-2)$th aircraft land on the same runway:

$$e_{j+3}^T Y \tilde{T}_{d\text{sep}} Y^T e_{j+1}^T - l_{j-1} \leq l_j \quad \text{for } j = 0, 1, \ldots, N-2 \tag{13}$$

The constraints for matrix Y are

$$\begin{cases} y_{i1} + y_{i2} + \cdots + y_{i2(N+1)} = 1 \\ y_{1(2j-1)} + y_{2(2i-1)} + \cdots + y_{(N+1)(2i-1)} \\ +y_{1(2i)} + y_{2(2i)} + \cdots + y_{(N+1)(2i)} = 1 \end{cases}$$
$$\text{for } i = 1, \ldots, 2(N+1) \tag{14}$$

with the addition constraint placed on y_{11} and y_{12} based on the designate landing runway for the boundary aircraft.

V. Minimizing Changes of STA from ETA

As shown in Fig. 6, due to the separation constraints, the changes of STA from ETA may become larger, and the larger the changes, the higher the cost for these aircraft [3]. Therefore, aside from the optimal operation-time considerations, we should also pay attention to minimizing the change of STA from ETA. The proposed approach here is to apply speed changes to these aircraft to minimize the changes of STA from ETA. Some aircraft may need to land ahead of their designed arrival time so that the summed changes of STA from ETA of all the aircraft within the controlling time window is minimized. The idea is depicted in Fig. 7.

A. Objective Function

In this section, our purpose is to minimize the changes of STA from ETA. The change of STA from ETA of the jth aircraft is

$$f_j = \left| \Delta t - \frac{v_j \Delta t}{v_{\text{new}(j)}} \right| \tag{15}$$

where Δt represents the width of the controlling time window. We will schedule the aircraft's velocity within this time window located in the en route sector. The modified flight time is decided by the new assigned speed $v_{\text{new}(j)}$. The goal is to minimize the change of STA from ETA for all the aircraft, which equivalently minimizes

$$F_2 = \Delta t^2 \sum_{j=1}^{N} \left(1 - \frac{v_j}{v_{\text{new}(j)}} \right)^2 \tag{16}$$

Note that speed increases can incur different fuel consumptions from speed reductions. To consider this difference, the cost function (16) can be modified as

$$F_2 = \sum_{j=1}^{N} h_j \tag{17}$$

and additional constraints can be added:

$$\begin{cases} \alpha_1 f_j \leq h_j \\ -\alpha_2 f_j \leq h_j \end{cases} \quad \text{for } j = 1, \ldots, N \tag{18}$$

where α_1 and α_2 are the cost weightings for speed increase and speed reduction. For simplicity, in the following, we will use Eq. (16) as our objective function.

B. Constraints

This section describes the constraints for all the aircraft when minimizing the changes of STA from ETA. The constraints include separation-time constraints, velocity constraints, and boundary constraints. The separation-time constraints are defined as

$$\frac{\Delta t v_{j+1}}{v_{\text{new}(j+1)}} + a_{j,j+1} - \frac{\Delta t v_j}{v_{\text{new}(j)}} \geq l_j \quad \text{for } j = 0, \ldots, N-1 \tag{19}$$

which means that the separation time must be big enough to meet the definition in Table 2 after applying the suggested speed. Since we are not controlling the boundary aircraft, we let $v_{\text{new}(0)} = v_0$. The time difference between the former and the later aircraft entering the controlling time windows is $a_{j,j+1}$. This value can be computed using

$$a_{j,j+1} = [e_{j+1} - e_j]^T S \tilde{e}$$

The velocity constraints can be defined as

$$v_j - v_{l(j)} \leq v_{\text{new}(j)} \leq v_j + v_{u(j)} \quad \text{for } j = 1, \ldots, N \tag{26}$$

which means that the suggested velocity must conform to the aircraft's performance specification.

The proposed formulation can be converted to a quadratic programming problem by defining the decision variable as

$$x_j = \frac{v_j}{v_{\text{new}(j)}} \quad \text{for } j = 0, \ldots, N \tag{21}$$

Then we have the following quadratic programming problem:

$$\min \quad f(x) = \Delta t^2 \sum_{j=1}^{N} (1 - x)^2$$

$$\text{subject to} \quad \begin{cases} x_j - x_{j-1} \geq \frac{l_{j-1} - a_{j-1,j}}{\Delta t} \\ -x_j + \frac{v_j}{v_j + v_{u(j)}} \leq 0 \\ x_j - \frac{v_j}{v_j - v_{l(j)}} \leq 0 \end{cases} \quad \text{for } j = 1, \ldots, N \tag{22}$$

The suggested velocity $v_{\text{new}(j)}$ can then be obtained from solving x_j. Then the aircraft in the controlling time window may use the suggested velocity and the suggested sequence to get the minimum changes of STA from ETA.

VI. Combined Optimization

The optimization problems presented in Secs. IV and V should not be solved independently. Using only the formulation in Sec. IV will probably get a solution with minimum operation time, but it has

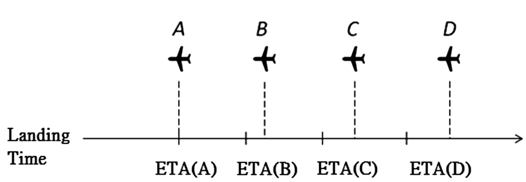

Fig. 6 Changes of STA from ETA.

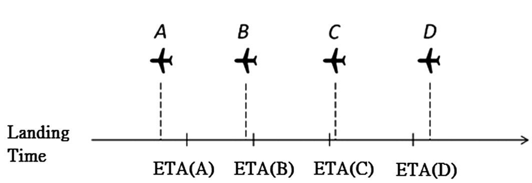

Fig. 7 Minimizing the changes of STA from ETA.

excessive changes of STA from ETA. The formulation presented in Sec. III cannot be solved without knowing the optimal sequence. Therefore, these two optimization problems need to be combined into a quadratic-constraint quadratic programming problem.

A. Quadratic-Constraint Quadratic Programming

By collecting the formulations in Secs. IV and V, we have the following combined optimization problem for a single-runway problem:

$$\min_{S,x_j} \quad w_1\left(\sum_{j=0}^{N-1} l_j\right) + w_2\left(\Delta t^2 \sum_{j=1}^{N}(1-x_j)^2\right) \tag{23}$$

$$\text{subject to} \quad \begin{cases} e_{j+1}^T Sr \le \tilde{e}_0 + \sum_{i=0}^{j-1} l_i \\ \tilde{e}_0 + \sum_{i=0}^{j-1} l_i \le e_{j+1}^T Sp \\ e_{j+1}^T ST_{\text{sep}} S^T e_j^T \le l_{j-1} \\ x_j - x_{j-1} \ge \frac{l_{j-1} - a_{j-1,j}}{\Delta t} \\ -x_j + \frac{v_j}{v_j + v_{u(j)}} \le 0 \\ x_j - \frac{v_j}{v_j - v_{l(j)}} \le 0 \end{cases} \quad \text{for } j = 1, \dots, N$$

$$\begin{cases} s_{11} = 1 \\ s_{i1} + s_{i2} + \ldots + s_{i(N+1)} = 1 \qquad \text{for } i = 1, \dots, N+1 \\ s_{1i} + s_{2i} + \ldots + s_{(N+1)i} = 1 \end{cases}$$
$$s_{ij} \in \{0, 1\} \tag{24}$$

Here, w_1 and w_2 are weighting numbers for adjusting the emphases placed on minimizing the operation time and the changes of STA from ETA. The objective function is linear in the variables s_{ij} and quadratic in the variables x_j. The constraints are either linear or quadratic in the decision variables. Hence, the combined problem is a quadratic-constraint quadratic programming problem. The case for parallel runway approaches can be formulated similarly.

B. Reducing the Calculation Time

By reducing unnecessary variables in the proposed formulation, a reduction in the calculation time is possible. Clearly, the elements in the sequence matrix S contribute the most to the number of decision variables. A careful examination of the possible sequences may eliminate some operationally unrealistic combinations. Since the STA for each aircraft should be located in its respective arrival time window, some sequence changes are unlikely to happen. Therefore, if the aircraft are presorted according to their ETA, a banded sequence matrix S of width m is adequate for representing all the possible optimal solutions. After removing unnecessary variables, $(N+1)^2$ variables in S will be reduced to $(2m+1)(N+1) - m(m+1)$ variables for $N \ge m$. Using this technique, the number of variables grows linearly with the number of aircraft. Consequently, the solution time required can be further reduced.

VII. Simulation Results

A. Signal-Runway Simulation

This section simulates the arrival sequence of aircraft using the historical data from Taiwan Taoyuan International Airport on 19 December 2008. We do not consider the mixed problems regarding takeoff and landing. The following simulations are run on an Intel Core 2 Duo 2.66 GHz PC with 2 GB RAM and solved using MATLAB and TOMLAB /CPLEX software packages.

Table 3 shows the radar data used in this simulation. The first column is the flight ID (where A/C denotes aircraft), and the second column is the ETA, which means how long it will take the aircraft to arrive at the airport relative to the first aircraft, JAL653, which is considered to be the boundary aircraft. The third column is the flight speed, and the fourth column lists the classification of each aircraft according to the definition in Table 2. Assume the allowable difference between the earliest and the latest arrival time is 5 min for each aircraft, and let the controlling time-window length be 70 min. Here, we put in two imaginary aircraft, aircraft A and aircraft B, to the historical data to make the situation more complicated. After introducing the initial conditions listed in Table 3, we get the optimal sequence listed in Table 4.

Using the optimal sequence, we obtain the x value for each aircraft as

$$x = [0.9924 \quad 0.9781 \quad 1.0005 \quad 0.9931 \quad 0.9548 \quad 1.0410 \quad 1.0379 \quad 1.0222]$$

Using the x value, we obtain the suggested velocity for each flight listed in Table 5. The ETA and STA for the arriving aircraft are listed in Table 6. The total changes of STA from ETA is 853 s. Compared

Table 3 Simulated arrival data

Flight ID	ETA, s	Velocity, kt	Classification
JAL653	0	420	Heavy
EVA806	200	590	Heavy
NWA69	282	370	Heavy
A/C A	320	450	Medium
CAL5215	387	370	Heavy
A/C B	430	450	Medium
EVA1757	506	500	Heavy
TNA356	670	570	Medium
EVA159	678	370	Heavy

Table 4 Result of the optimal sequence

Sequence	Original sequence	Optimal sequence
1	EVA806	EVA806
2	NWA69	NWA69
3	A/C A	A/C A
4	CAL5215	A/C B
5	A/C B	TNA356
6	EVA1757	CAL5215
7	TNA356	EVA1757
8	EVA159	EVA159

Table 5 Suggested velocity of the aircraft

Flight ID	Original speed, kt	Optimal speed, kt
EVA806	590	607
NWA69	370	378
A/C A	450	450
CAL5215	370	355
A/C B	450	453
EVA1757	500	482
TNA356	570	597
EVA159	370	362

Table 6 Original arrival time and optimized arrival time

Flight ID	ETA, s	FCFS STA, s	Optimal STA, s
EVA806	200	200	84
NWA69	282	306	190
A/C A	320	438	322
CAL5215	387	517	559
A/C B	430	649	401
EVA1757	506	728	665
TNA356	670	860	480
EVA159	678	939	771

with the changes of STA from ETA using FCFS method, which is 1075 s, we have a 20.7% reduction in the changes of STA from ETA.

B. Parallel-Runway Simulation

There are two parallel runways (05/23 and 06/24) at Taiwan Taoyuan International Airport. Let runway 05 be runway L, and let runway 06 be runway R. Because the distance between the runway centerlines is 1505 m, which does not meet the independent operation standard of the ICAO regulation, the runways in this airport should be operated using a dependent operation. According to [19], the minimal diagonal distance-based separation is 2 n mile. This distance is transformed to a time-based separation by using Eq. (12). We use the data in Table 7 to simulate the optimal sequence for the arriving aircraft landing on parallel runways.

Using the proposed algorithm, we get the optimal results listed in Table 8. Note that HVN924 is now able to land before CAL005. Using the optimal sequence, we get the x value for each aircraft as

$$x = [\,0.9867 \quad 0.9902 \quad 1.0017 \quad 1.0100 \quad 1.0131\,]$$

The suggested speeds for each aircraft are listed in Table 9. The ETA and STA for these arrival aircraft are listed in Table 10. Using the optimal flight speed, the changes of STA from ETA is 194 s. Compared with the result using FCFS method, which is 224 s, we have a 13.4% reduction in the changes of STA from ETA.

C. Monte Carlo Simulation

Now we compare the proposed algorithm with the FCFS algorithm using Monte Carlo simulation. In this simulation, we generate 15 aircraft with random mixture of heavy and medium types. The ETA of each aircraft is generated from a uniform distribution between 0 and 1500 s. The speed of each aircraft is generated from a uniform distribution between 369 and 589 kt. Figure 8 shows the computational results for comparing the operation time. The black line represents the total time for landing all the aircraft using optimal scheduling. The gray line represents the results using the FCFS algorithm. The horizontal coordinate represents the number of simulations. The vertical coordinate represents the operation time used. From Fig. 8, if we use the optimal scheduling, the average improvement is about 5%. The result for comparing the changes of STA from ETA is shown in Fig. 9. It is clear that the proposed algorithm is better than the FCFS algorithm.

Now we will use different widths of controlling time windows to test the scheduling efficiency. Table 11 shows the average operation-time improvements for different widths of controlling time window.

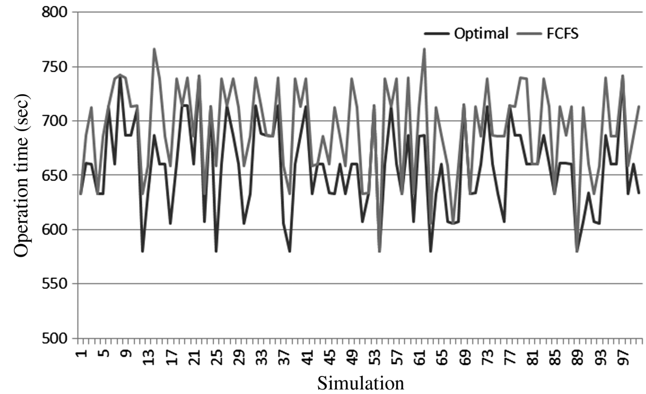

Fig. 8 Comparison of the FCFS operation time and optimal operation time.

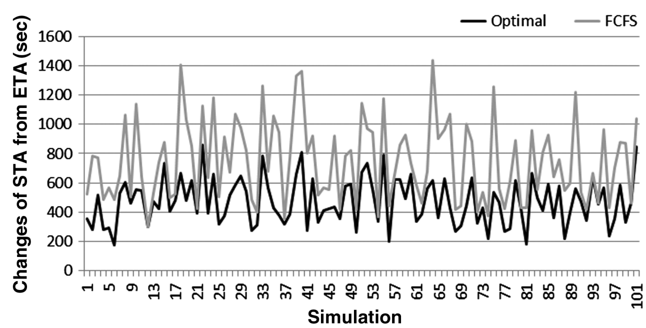

Fig. 9 Comparison of the FCFS time variation and optimal time variation.

Table 7 Simplified arrival data

Flight ID	EST, s	Velocity, kt	Type	Classification
EVA238	0	332	B763	Heavy
CAL762	200	590	B744	Heavy
CAL005	282	370	B744	Heavy
HVN924	387	370	B744	Heavy
JAL653	506	500	B777	Heavy

Table 8 Final result of the sequence

Sequence	Runway selection	Optimal sequence
1	06	CAL762
2	05	HVN924
3	06	CAL005
4	05	JAL653

Table 9 Suggested speed of the aircraft

Flight ID	Original speed, kt	Optimal speed, kt
CAL762	480	486
CAL005	385	381
HVN924	570	576
JAL653	420	415

Table 10 Comparison of FCFS method and the proposed method

Flight ID	ETA, s	FCFS STA, s	Optimal STA, s
CAL762	225	225	169
CAL005	233	273	275
HVN924	259	357	405
JAL653	269	405	324

Table 11 Effect of controlling time-window width with regard to operation time

Width of controlling time windows, min	Average operation time	Average efficiency, %
90	648.76	5.73
80	657.53	5.39
70	663.40	5.18
60	668.26	4.61

Table 12 Effect of controlling time-window width with regard to changes of STA from ETA

Width of controlling time windows, min	Average changes of STA from ETA	Average efficiency, %
90	357.54	43.57
80	365.11	40.53
70	373.50	37.76
60	382.35	33.83

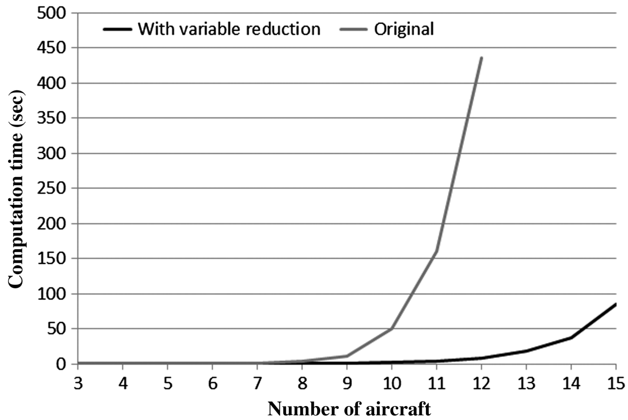

Fig. 10 Calculation time for different numbers of aircraft (signal runway).

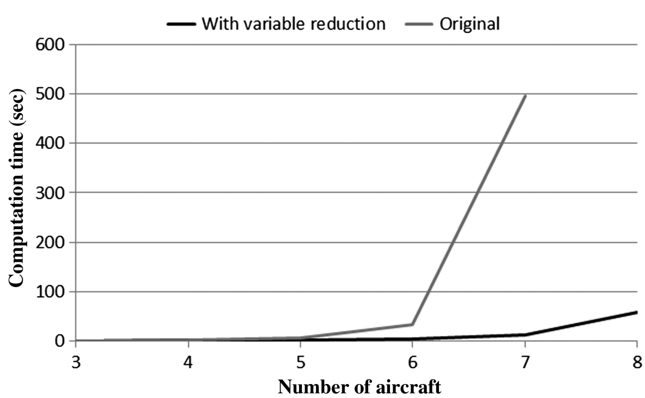

Fig. 11 Calculation time for different numbers of aircraft (parallel runway).

The efficiency shows the percentage of reduction in operation time, compared with the FCFS method. In Table 12, we compare the changes of STA from ETA of the FCFS algorithm and the proposed optimization formulation. From these two tables, the longer the width of the controlling time window, the higher the improvements we can get. However, a longer width may include more aircraft in the formulation, and computation time will increase accordingly. Therefore, a proper selection of the controlling time-window width is essential to efficiently applying the proposed algorithm.

D. Reducing the Calculation Time

In this section, we compare the computation time used with and without using the variable-reduction technique. Figure 10 is the computation time for different numbers of aircraft arriving at a single-runway airport. Figure 11 is the computation time for different numbers of aircraft for a parallel-runway case. The horizontal coordinate represents the number of aircraft. The vertical coordinate represents the computation time used. In these figures, it is clear that the computation time grows rapidly when the number of aircraft increases. However, using the variable-reduction technique, the computation time can be greatly decreased.

VII. Conclusions

When aircraft arrive at an airport, proper separations between aircraft are required. The proposed algorithm first checks if the separation time is sufficient using the ETA of all the aircraft being considered. The arrival time separation should meet ICAO regulations. If the system detects conflicts, the proposed algorithm generates optimal sequences that have minimal operation times for all the arriving aircraft. In conjunction with minimizing the operation time, the changes of STA from ETA can also be minimized by using the proposed algorithm. Much of the existing research has been focused on scheduling aircraft in the terminal area. If the number of the arriving aircraft is beyond the capacity of the airport, the terminal area may become crowded. This paper proposes an algorithm to control the speed of aircraft in the en route sector. As a result, these aircraft can have proper separations before entering the crowded terminal area. The proposed algorithm uses the idea of sequence matrix formulation so that the constraint number is greatly reduced. With the additional variable-reduction technique, the proposed algorithm can solve larger problems using fewer computation time.

In the proposed algorithm, we consider the runway as the only merging point for all the arriving aircraft. In reality, aircraft may need to pass several merging points before landing. Therefore, conflicts may occur before aircraft reaching the runway. The separation at multiple merging points should also be considered in order to make the proposed algorithm fit into more realistic situations. Moreover, uncertainties of the ETA of the aircraft are not considered. Separation time may increase under the existence of ETA uncertainties. The extension of the proposed model to a model with multiple merging points and surveillance errors will be pursued in the future work.

Acknowledgment

This work is supported by National Science Council under grant NSC-96-2218-E-006-283-MY3, which is greatly appreciated.

References

[1] "International Air Transport Association (IATA) Response to the Commission for Aviation Regulation's (CAR) Draft Determination CP 3/2009 of 18 June 2009 on the Maximum Levels of Airport Changes at Dublin Airport," International Air Transport Association, Montreal, 2009.

[2] Bianco, L., Dell'Olmo, P., and Odoni, A. R., *Modelling and Simulation in Air Traffic Management*, Springer–Verlag, Berlin, 1997.

[3] Beasley, J. E., Krishnamoorthy, M., Sharaiha, Y., and Abramson, D., "Scheduling Aircraft Landings—The Static Case," *Transportation Science*, Vol. 34, No. 2, May 2000, pp. 180–197. doi:10.1287/trsc.34.2.180.12302

[4] Denery, D. G., and Heinz, E., "The Center-TraCON Automation System: Simulation and Field Testing," NASA Ames Research Center, Moffett Field, CA, 1995.

[5] Davis, T. J., Krzeczowski, K. J., and Bergh, C., "The Final Approach Spacing Tool," *Proceedings of the 13th IFAC Symposium on Automatic Control in Aerospace*, 1994.

[6] Helmke, H., Hann, R., and Uebbing-Rumke, M., Müller, D., and Wittkowski, D., "Time-Based Arrival Management for Dual Threshold Operation and Continuous Descent Approaches," *Eighth USA/Europe Air Traffic Management Research and Development Seminar*, 2009.

[7] Dugail, D., Feron, E., and Bilimoria, K., "Stability of Intersecting Aircraft Flows Using Heading Change Maneuvers for Conflict Avoidance," *Proceedings of the American Control Conference*, 2002.

[8] Mao, Z. H., Feron, E., and Bilimoria, K., "Stability and Performance of Intersecting Aircraft Flows Under Decentralized Conflict Avoidance Rule," *IEEE Transactions on Intelligent Transportation Systems*, Vol. 2, No. 2, June 2001, pp. 101–109. doi:10.1109/6979.928721

[9] Krozel, J., and Peters, M., "Strategic Conflict Detection and Resolution for Free Flight," *Proceedings of the 36th Conference on Decision and Control*, 1997.

[10] Abela, J., Abramson, D., Krishnamoorthy, M., Silva, A. D., and Mills, G., "Computing Optimal Schedules for Landing Aircraft," *The 12th National Conference of the Australian Society for Operations Research*, July 1993.

[11] Beasley, J. E., Krishnamoorthy, M., Sharaiha, Y. M., and Abramson, D., "Displacement Problems and Dynamically Scheduling Aircraft Landings," *Journal of the Operational Research Society*, Vol. 55, No. 1, 2004, pp. 54–64. doi:10.1057/palgrave.jors.2601650

[12] Zhang, X., Zhand, X., Zhang, J., and Liu, B., "Optimization of Sequencing for Airport Arrival Based on Approach Routes," *Proceedings of the 2007 IEEE Intelligent Transportation Systems Conference*, Inst. of Electrical and Electronics Engineers, Piscataway, NJ, 2007, pp. 592–596.

[13] Resmerita, S., and Heymann, M., "Conflict Resolution in Multi-Agent Systems," *Proceedings of the 42nd IEEE Conference on Decision and*

Control, Inst. of Electrical and Electronics Engineers, Piscataway, NJ, 2003, pp. 2537–2542.
[14] Lin, C. E., and Jian, S. G., "A Fast Approach Queuing Observer for Flow Control Using FDP in Excel Environment," *Journal of Aeronautics, Astronautics, and Aviation*, Vol. 40, No. 1, 2008, pp. 1–10.
[15] Kupfer, M., "Scheduling Aircraft Landings to Closely Spaced Parallel Runways," *Eighth USA/Europe Air Traffic Management Research and Development Seminar*, 2009.
[16] "Air Traffic Management Procedures," Civil Aeronautics Administration, ROC, 2007.
[17] Frevile, E., and Nicolaon, J.-P., "Potential Benefits of a Time-Based Separation Procedure to Maintain the Arrival Capacity of an Airport in Strong Head-Wind Conditions," *Fifth USA/Europe Air Traffic Management Research and Development Seminar*, 2003.
[18] Janic, M., "Toward Time-Based Separation Rules for Landing Aircraft," *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 2052, 2008, pp. 79–89. doi:10.3141/2052-10.
[19] "Rules of the Air and Air Traffic Services," International Civil Aviation Organization, Doc 4444-RAC/501, Montreal, 1996.
[20] "AIP Taipei FIR," Civil Aeronautics Administration, ROC, 2008.
[21] *Air Traffic Control*, Federal Aviation Administration, Washington, D.C., 2010.